MINOR STUDIES FROM THE PSYCHOLOGICAL LABORATORY OF CLARK UNIVERSITY.

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XIX. A PRELIMINARY STUDY OF THE PSYCHOLOGY OF REASONING.

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An examination of the treatment of reasoning by most psychological writers reveals the fact that they have been largely influenced by the demands of formal logic. For many centuries the syllogism was regarded as the only form of reasoning. So recent a writer as John Stuart Mill enters a vigorous protest against the attempt, on the part of a few, to look on the syllogism as useless in reasoning. For him the syllogistic form is absolutely indispensable in testing the correctness of generalizations. But he admits that the syllogism is not a correct analysis of the psychic process of reasoning.

According to Mercier the regard for the syllogism, as a form of thought, has declined during the last thirty years. He affirms that the study of the reasoning processes should form a part of psychology; but he admits that in his discussion of the matter he has "bowed the knee in the House of Rimmon."

Lloyd Morgan declares that only those beings reason that are capable of "focusing the therefore," and implies that reasoning is a conscious attempt to justify our conclusions. It is an examination of the method by means of which we reach a conclusion. William James is still willing to say that "reasoning may be very well defined as the substitution of parts and their implications or consequences for wholes," though he treats the matter at some length descriptively.

Binet finds that there are three images involved in reasoning. The first calls forth the second by resemblance. The second suggests the third by contiguity; and this is all that there is in reasoning. Ribot would have us believe that "reason is only a means for control and proof."

George M. Stratton, however, writing in the *Psychological Review* for 1896 maintains that both logic and psychology have an interest in reasoning. But in the past the part that belongs to each science has not been very well made out and much confusion has resulted. Each has a different end in view and

employs a different method. In brief, Stratton holds, and rightly, that in logic, there is an attempt to justify the conclusion; in psychology there is an attempt to explain how the conclusion was reached.

These few and brief citations will serve to show how the discussion of reasoning has been influenced by formal logic even in the case of recent writers on the subject.

But little attention has been given to an experimental study of the question as yet. This may be explained in part by the fact that sensation and perception have furnished the experimentalist easier points of attack. Indeed, some have gone so far as to assert that reasoning is such a very complex process that we have little or no chance of ever being able to catch the mind in the act of pure reasoning. Groos, who has made some attempt in this direction, thinks that perhaps we shall be able to get most empirical material by noting the blunders of reasoning into which we fall. That reasoning is a difficult matter to deal with empirically is certainly evident to all, but it does not seem fair to place it among the impossible problems without further consideration.

The writer confesses himself more hopeful of a solution of these problems by the empirical method. Certain phases of reasoning may be observed in the solving of all problems and puzzles, and by introspection it ought to be possible to gain a fair account of the psychic process involved in the reasoning act. Believing that some information concerning reasoning could be obtained in this way, two series of simple experiments have been carried out.

The First Series of Experiments.

The first set of experiments was made with a problem in long division, found in the "puzzle column" of the *Congregationalist*.

The problem is as follows:

```
9 x x ) 4 x x x 4 x 7 ( x x x x
          x 9 x x
          -------
            x 1 x x
            4 x x 5
            -------
              2 x 7 x
              x x x 4
              -------
                x 0 x
```

It was the task of the subject to supply the proper figures for the crosses, and to take note, so far as he was able, of his mental processes while doing so. These introspections were recorded by an operator who was present during the solution. In this way it was possible to secure a somewhat full account of what went on in the mind during the solution. The problem was solved in this way by twenty-six persons.[1]

The following imaginary record is typical of the subjects, method of procedure except that the actual ones were less certain and direct. All notes and introspections also are here omitted. The procedure may be followed in the example below where the large figures are those of the original problem, the small figures above them are the numbers of the positions of the individual figures and x's numbered consecutively from left to right, and the letters below indicate the successive steps of the solution.

```
1  2  3     4  5  6  7  8  9  10     11 12 13 14
9  x  x  )  4  x  x  x  4  x  7   (  x  x  x  x
   n  h        r  r  r     f         p  g  h  j
   o  l  )  15 16 17 18             (q     i  k
            x  9  x  x
            -----------------
                  19 20 21 22
                  x  1  x  x
                  m     b  e
                  23 24 25 26
                  4  x  x  5
                  --------------
                     27 28 29 30
                     2  x  7  x
                              a
                     31 32 33 34
                     x  x  x  4
                           d
                     -----------
                        35 36 37
                        x  0  x
                              c
```

a. Inserted 7 in *30* from data given.
b. Inserted 4 in *21* from data given.
c. Obtained 3 in *37* by substraction.
d. Obtained 7 in *33* by substraction.
e. Obtained 2 in *22* by general rule of subtraction.
f. Inserted 2 in *9* from data found in fifth step.
g. Inserted 0 in *12* from data found in *21* and *22*.
h. Decided that *3* and *13* must be odd because *26* is odd.
i. Decided that *13* must be 5 because *23* is 4.
j. Decided that *14* must be 2 or 3 because of *27*.

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[1]The writer wishes to express his obligation to Dr. Alvin Borgquist for the notes upon these solutions which were taken by him under the direction of Dr. Sanford.

k. Decided that *14* must be 2 because *3* is odd and *34* is even.

l. Decided that *3* must be 7 because it is the only odd number that multiplied by 2 and 5 gives 4 and 5 in units place in the product.

m. Decided that *19* must be 5 because *24* is greater than 1.

n. Decided that *2* should be 3 or 8 by alteratives.

o. Decided that *2* must be 8 by trial and error.

p. Decided that *11* must be 4 or 5 because of *4*.

q. Decided that *11* must be 5 by trial and error.

r. Inserted 9 8 6 in *5 6 7*.

A few other relations were used by a few of the subjects. The most important is the discovery that the 24th figure is 9.

The subjects tested in this series were of very various degrees of skill in introspection and it is quite clear that most were unable to do more than indicate the steps taken without entering upon a minute account of them. This is not at all remarkable, however, for there was apparently nothing but the associative rise of the required numbers in consciousness that could have been observed. The writer's own experience with the problem, confirmed by that of the best expert subjects that have been tested, seems to show that the reasoning processes involved are matters of simple associative recall. The particular feature of the problem which engages attention at the moment calls up associatively the digit required, or, when the case is more complicated, the several possibilities. *No trace appears of anything other than the familiar processes of association and apperception working under the special conditions of attention to the particular matter in hand.*

Of more importance perhaps than the elementary functions involved in reasoning, is the general method of attack upon the problem presented, the way in which clues are looked for and the adequacy with which they are apprehended when found. Here lies an immense field for individual differences. After tabulating the solutions of the problem, it was found possible to group them with regard to efficiency in three classes. The basis for the classification is the number of steps required to reach the solution and for this reason is arbitrary.

The best group consisted of seven solutions: 2 requiring fourteen steps; 1 fifteen; 1 sixteen; 1 seventeen and 2 eighteen. The medium group consisted of eleven solutions: 2 of nineteen steps; 3 of twenty; 2 of twenty-one; 3 of twenty-two and 1 of twenty-three. The poorest group consisted of eight solutions: 2 of twenty-four steps; 1 of twenty-five; 1 of twenty-six; 1 of twenty-eight; 1 of thirty-four; 1 of thirty-five and 1 of forty-nine.

The difference in the number of steps required to solve the

problem shows the individual differences. It is no doubt true that the variation in the number of steps may be explained in part by the difference in completeness with which the introspections were made. But some of the variation must certainly be attributed to psychical factors, such as methods of association, sagacity, and familiarity with the particular numerical relations involved. It is to be noted, however, that those most skilled in mathematics did not always reach the solution by the use of the fewest steps.

The different groups show different amounts of mental fumbling. There is, as might be expected, a close relation between the amount of fumbling and the number of steps required since the fumbling, or inability to see essential relations clearly, is the chief cause of the lengthening of the process of solution. When the successive steps of the solution are treated in groups of five the earlier groups show a higher percentage of fumbling than the later ones. There is much fumbling until a cue is found, which enables the subject to take a few definite steps. This, however, is soon exhausted and there often must then be another search with renewed fumbling. Only a few relations are simultaneously before the mind.

The method of attack seems to be influenced by habit. Five out of seven in the best group of subjects began with the first figure of the quotient. Seven out of eleven in the second group began with the same figure, while only three out of eight in the third group began with this figure. These facts show a rather strong tendency to follow the fixed habits in commencing the solution, and a certain advantage in so doing, or in the methodical habits of procedure that it indicates. They begin at the point where one would in solving an ordinary problem in division. It is in the formation of such habits that growing skill in mental operations consists.

THE SECOND SERIES OF EXPERIMENTS.

In the second series of experiments the material was of the same nature as before, but the problems were much simpler. The form of each problem follows:

I.	II.	III.	IV.
$\text{x x 7}\,)\,2077\,(2$	8	$\text{x}\,)\,\underline{\text{x}\,6\,4}$	$6\,\text{x}$
$\underline{\text{x x x}\,4}$	x	$1\,2\,4$	$\underline{\text{x}}$
$1\,0\,3$	$\underline{5}$		$5\,9\,4$
	$1\,8$		

V.	VI.	VII.
$\begin{array}{r} 76 \\ \underline{5x} \\ 17 \end{array}$	$\begin{array}{r} x\,)\,\underline{x6x6} \\ 1924 \end{array}$	$\begin{array}{r} x74 \\ 2x4 \\ \underline{35x} \\ x532 \end{array}$

The task in these problems was the same as in the previous one. In all twenty persons have solved these seven problems and given a rather full account of the mental processes taking place during the solution. These subjects were all more or less experienced in introspection. An operator was present at the time the work was done and secured as full an account of the work as possible. The direct reports as to the detail of the processes was as meagre as before, but the simpler conditions permit more certain inferences as to its character. For an indication of the character of the data obtained, see pp. 499-501 below.

On examining the results, it was found possible to group the solutions according to the following scheme.

TABLE I.

Problems.	I.	II.	III.	IV.	V.	VI.	VII.
Sub- 1	B	A	E	B	C	D	C
ject 2	B	A	E	B	C	E	C
" 3	A	A	D	A	A	D	C
" 4	B	A	E	B	C	E	C
" 5	B	A	E	B	C	D	C
" 6	B	A	E	B	C	D	C
" 7	B	C	D	B	C	D	C
" 8	B	C	E	B	C	E	C
" 9	B	A	E	A	A	E	C
" 10	B	A	E	A	C	E	C
" 11	A	A	D	A	C	D	C
" 12	B	A	D	A	A	D	C
" 13	B	A	E	A	C	E	A
" 14	B	A	E	A	C	E	C
" 15	B	A	E	A	C	E	C
" 16	A	A	D	B	A	E	C
" 17	A	A	E	A	A	E	C
" 18	B	C	E	B	C	E	C
" 19	B	C	E	B	C	D	C
" 20	B	A	E	A	A	D	A

Explanation of letters used in classifying the solutions.

Problem I.

A = The use of a general principle. This involved the subtracting of the remainder from the dividend and dividing by the quotient.

B = Trial and error method. The attempt to supply the missing figures by comparing the quotient and the divisor. The process was not wholly dependent on chance, but involved more or less uncertainty.

Problem II.

A = The use of a general principle. This consisted in adding 5 and 8, and subtracting the sum from 18.

C = The missing figure was supplied by mechanical sustitution.

Problem III.

D = The problem was solved by using one variable. The divisor was used as multiplier of the quotient. The divisor was changed until a product was obtained with a 6 in the ten's place.

E = The problem was solved by using two variable terms,—the divisor and the first figure in the dividend.

Problem IV.

A = The use of a general principle. This was done by dividing the product by the multiplicand.

B = The trial and error method. This consisted in assuming values for the missing figures and changing them until the conditions were satisfied.

Problem V.

A = The use of a general principle. The remainder was subtracted from the minuend.

C = Mechanical substitution as the result of familiarity with the relations involved.

Problem VI.

D = The use of one variable.

E = The use of two variables.

Problem VII.

A = General principle used. See A under the second problem.

C = Mechanical substitution. See C above.

Methods *D* and *E* stand between methods *A* and *B*, in that they are guided by general principles though not such as can be so easily formulated and brought definitely to consciousness, and even in the *B* method the trials are not wholly at random.

Summary of the table.

Problem I.	Problem II.	Problem III.	Problem IV.
A = 4	A = 16	E = 15	A = 10
B = 16	C = 4	D = 5	B = 10

Problem V.	Problem VI.	Problem VII.
A = 6	D = 9	A = 2
C = 14	E = 11	C = 18

Problem I was to many rather difficult, and the results show a very strong tendency to use the "trial and error" method in the solution. This method was persisted in in spite of the fact that a very direct method was plainly indicated by the data given. The form of the problem seems to have suggested old associations and the subject at once began the search for the cue by using the quotient as one factor. It was difficult for most to change this first impression and often a struggle resulted. Those who applied the general principle did so at once and the result followed immediately.

The second problem was an easy one and these subjects show

a strong tendency to use a general principle in its solution. The solution followed almost as soon as the problem was perceived, but fifteen out of twenty testified that a solution was reached by applying a general principle. Possibly, however, the general principle may have been appealed to for justification of the result to themselves rather than in reaching the solution.

The third problem was more difficult, but there was an increase in the number of those who attacked the problem in the most direct way. In this problem and in the sixth one the most direct method was to use but one variable quantity. It is noted that there is an increase in the number of those who saw the most direct method of solving this problem. This may be due to a "warming up" to the work, to the influence of practice, or to the refreshing the mind for the old numerical relation.

The fourth problem was selected because it presented conditions of medium difficulty. The subjects fall into two equal classes. Those who used the *A* method applied the most general principle possible to its solution. Those who used the *B* method followed the "trial and error method." They varied the two missing figures until a set was found that satisfied the conditions.

In the fifth problem fourteen out of twenty found themselves applying a general principle of subtraction. They subtracted the remainder from the minuend and thus obtained the missing figure in the subtrahend. Some felt sure, however, that the missing figure was supplied almost spontaneously on seeing the problem.

The seventh problem contained very simple conditions and the results show that eighteen out of twenty saw the missing figures instantly on seeing the given figures in each column. These eighteen assert that there was no adding of the given figures and subtracting this sum from the grand total. The associations were so ready that they resulted at once on seeing the conditions.

Some General Observations Made by the Subjects.

One subject reports that a solution which starts off with success but breaks down at a point near the end, produces something like a mental "cramp." The mind is reluctant to abandon a course that has almost resulted in success. There is a tendency to believe that the data are wrong rather than admit there is an error in the computation. There seems to be a poverty of possible chances, and a tendency to force a solution along lines that give an initial success. The same person remarks that he tries to solve puzzles by trying the same plan time after time. An initial success which proves to be partial

makes it all the more difficult to modify a plan of attack, while initial failure leads more readily to new plans.

The fourth problem is as follows:

$$\begin{array}{r} 6\,x \\ x \\ \hline 594 \end{array}$$

One subject noticed that the fact that a six was placed in ten's position in the multiplicand caused him to conclude that the missing figure in the multiplicand cannot be a six. The mere presence of the six had excluded the possibility of the missing figure being a six. He had a feeling that the missing numbers must be eight and eight, and it was difficult to drive himself into an admission that the missing numbers must be nine and six. The thought of putting another six in the multiplicand was certainly inhibited by the presence of one six already in the multiplicand. Several others noticed that they were led to believe that the missing figures in the multiplicand and multiplier were alike. The fact that each was represented by the same sign seemed to suggest this to them, although they had been told at the outset that a cross may represent any one of the digits.

One subject reports that the large product in the fourth problem suggested a large digit for the multiplier. The subject was conscious that the 594 was very near to 600, and because of a tendency to work with ten as a unit the multiplier was seen to be 9.

Another testifies that the missing figures in the fourth problem were kept in the foreground, while the 6 and 59 were in the background.

Another reports that he tried 6 in the multiplier of the fourth problem because he felt that the multiplier must be a reasonably large number. The six was taken before he had considered all the data, and the conclusion seems to have been based on a vague consciousness of the relations existing between the given factors.

One person in solving the third problem asserts that he did not try a large number in the divisor at first, for the product obtained by multiplying the divisor by the quotient must not contain more than three figures. He must keep within this limit.

It may be suggested that the figures with which the subject began give a fair indication as to his general feeling regarding the numbers to be supplied.

The following will illustrate the nature of the data gathered:

The A Method in the First Problem.

The subject first saw that the second line contains blanks and at once resorted to a general rule of subtraction— "Given the minuend and the remainder to find the subtrahend." As soon as the subtraction was completed he knew that the second line was twice the divisor. This division gave the required divisor.

The B Method in the First Problem.

$$\begin{array}{cccccccccc}
\overset{1}{x} & \overset{2}{x} & \overset{3}{7} & \Big) & \overset{4}{2} & \overset{5}{0} & \overset{6}{7} & \overset{7}{7} & \Big(& \overset{8}{2} \\
 & & & & \overset{9}{x} & \overset{10}{x} & \overset{11}{x} & \overset{12}{4} & & \\
\cline{5-8}
 & & & & & \overset{13}{1} & \overset{14}{0} & \overset{15}{3} & &
\end{array}$$

The first idea was that the figure in position (1) may be 1 or 9. But it was soon decided to reject 9 on the account of 2 in position (4). At this point it was discovered that 2 was the entire quotient. The problem was begun anew by multiplying seven in position (3) by 2 in position (8). Since the figure in position (11) is 7, the figure in position (2) must be such that when multiplied by 2 will equal a number ending in 6 for a product. This will make the figure in position (2) equal 3. Then the first in the divisor is a 9 after all. Then 9, 8, 7, 6, 5 and 4 were tried as the second figure of the divisor, but all were rejected and 3 was declared once more to be the correct number for the position. But this does not work. At this point the subject was forced to repeat the series 3, 4, 5, 6, 7, 8 for position (2). By this repetition 8 was found to be the correct figure. In the first trial 8 seems to have been skipped in some way.

The A Method in the Second Problem.

The subject saw at once that the process was addition. Immediately he saw $8 + 5 = 13$. $18 - 13 = 5$. The first step came without hesitation and then there was a slight delay in the subtraction process, due perhaps to the verifying of the results.

The C Method in the Second Problem.

The sum was seen to be 18. $5 + 8 = 13$, and at this point saw that the number required equaled 5. He did not say "13 from 18" to get his 5.

The D Method in the Third Problem.

The subject began with 24 in the quotient and the six in the dividend. He noted that the first multiple of 24 containing 6 in the ten's place was 7. $7 \times 24 = 168$. Having done this he concluded that the divisor was 7. The entire dividend was then obtained by multiplying 124 by 7. It required only a few seconds to find the blanks, and the subject began at once his solution with 24 and 6 as the clew to the solution.

The E Method in the Third Problem.

$$x\ \big)\ \underline{x\ 6\ x} \\ \quad\ \ 1\ 2\ 4$$

The subject began by saying x into x will equal 1. But this told him nothing. He then looked at 6 in the dividend and 12 in the quotient and repeated $12 = 2 \times 6$. He dropped this clue as it did not amount to

anything. He then saw that six in the dividend was the clew. 2 into 6 equals 3. From this he decided that 3 was the divisor and three was the first figure in the dividend. Here he was balked. He then tried 4 in the divisor. Impossible to say just what was done here. He lingered over this point for some time. He then multiplied 124 by 2 to see if it would not help; but it would not do. He had already rejected 3 from the divisor. He then returned to the divisor and tried 2 once more forgetting that it had already been rejected. He then tried dividing 26 by 2 with 13 as the quotient. This did not help. He tries 3 in the divisor once more. "Must have felt that I made a mistake before." He then realized that he was trying each digit in succession. He now tried 4 but soon rejected it. He now felt that he did not want to follow up the scale any further. But at the same time this would give the correct result. He now returns to 124 in the quotient and multiplies it by 4. He was now ready to try anything to get a change of attack. He returned to 3 for the divisor once more, being influenced by the 2 in the quotient and the 6 in the dividend, although this had been worked out and the 3 rejected from the divisor. He now saw that 124 in the quotient would have to be multiplied by something so as to have 6 in the ten's place as shown by the 6 in the dividend. He then tried 5, 6 and seven as multipliers. 7 gave the desired result.

The A Method in the Fourth Problem.

As soon as the subject saw the data, he divided 594 by 6. This gave 9 for the multiplier. Then $9 \times 60 = 540$. $594-540 = 54$. 9 into 54 $= 6$, the missing figure in the multiplicand.

The B Method in the Fourth Problem.

The subject's first thought was that the missing figures must be alike and that 8×8 would satisfy the conditions. He had a feeling that the missing figure in the multiplicand must not be 6, for the 6 already in the multiplicand closed the door against the possibility of another 6 in the multiplicand. But 8×8 gave 64. This would not do. With 9 in the multiplier and six in the multiplicand the product would be 54. This showed that he must have 5 to carry. He then thought of 10×6. But he could scarcely get away from the 8×8. He was then pushed into $9 \times 6 = 54$. The 6 certainly inhibited the placing another 6 in the multiplicand.

The A Method in the Fifth Problem.

The subject saw at once that the process was subtraction. $76 - 17 = 59$. The method was suggested just as soon as he saw the data.

The C Method in the Fifth Problem.

The subject sought for a number which when subtracted from 16 would give 7 for a remainder. He saw that the missing number must be larger than 6 to give 7 for a remainder. He then added 10 to 6. Then he sought for a number which subtracted from 16 would equal 7. This number was 9. But he was not satisfied with 9 till the subtraction was completed.

The D Method in the Sixth Problem.

The process is division by a single number. There is no remainder. To secure 4 in the quotient the last 6 in the dividend must become a 16 by carrying one ten. Then 4 is tried in the divisor. The clue was

taken from the last figure in the dividend and the last figure in the quotient. The problem was solved backward.

The E Method in the Sixth Problem.

The subject saw that he must have a large remainder after the first division in order to have 9 as the second figure in the quotient. He first tried 3 for the divisor and 5 for the first figure in the dividend. This would not do. He then looked at the end and saw the divisor must be an even number or something that would divide a number ending in 6 evenly. He rejected 5 at once for the divisor. He then tried 4 in the divisor and 7 in the first place of the dividend. This verified.

The A Method in the Seventh Problem.

The subject saw that $4 + 4 = 8$. $12 - 8 = 4$. $5 + 7 + 1 = 13$. $13 - 13 = 0$. $3 + 2 + 1 = 6$. $15 - 6 = 9$.

The C Method in the Seventh Problem.

The subject saw at a glance that the missing figure in the first column must be 4. He said $4 + 4$ are 8 and $4 = 12$. Then said 7 and 5 and $1 = 13$. The missing figure in the second column is 0. Then added the last the same as the first.

In a general way the solutions are upon three levels.

1. *Direct substitution.* These subjects were so familiar with the different numerical combinations that the missing figure was brought to the front as soon as the data were apperceived. Some may wish to classify this as memory. But is it? In all probability this was the first time that these particular figures were ever given to the subject in exactly this relation. To the writer's mind it is the simplest or limiting case of reasoning. The series from it to the more complicated cases is an unbroken one, and the latter are, so far as I can discover, but compoundings of such simple associations. In this group come those solutions marked *C*.

2. *General Principle.* These subjects were not so facile with the mere numerical relations and were unable to reach the missing figures by direct association. They saw clearly enough the general principle involved but the supplying of the data had to take a slower course. The associative processes operated within limits imposed by the "general principle" apperceived. In this group come those marked *A* and *D*.

3. "*Trial and Error.*" The subjects using this method saw the relations less perfectly. They were not consciously guided by general principles but set about fulfilling the conditions empirically. The solution was often delayed and frequently the steps were repeated. In this group are found those solutions marked *B* and *E*.

It should not be concluded that the general reasoning powers of the subjects themselves are fully characterized by these tests. For these groups do not represent stages of develop-

ment The type selected for the solution of a problem was determined largely by the subject's familiarity with the conditions given. Perhaps all use the method of "trial and error" when a difficult problem is first attacked. DuBois-Reymond quotes Helmholtz who thus describes his mental processes in solving mathematico-physical problems:

"I must compare myself to a mountain climber, who, without knowing the way, mounts slowly and painfully. Often he must turn back for he can go no further. Sometimes through intuition, sometimes through accident he discovers a trace of a new way. This leads him forward again for a short distance and finally he reaches his goal. Then he discovers to his shame a royal way upon which he could have come, if he had only been clever enough to find the correct beginning."

General Conclusions.

1. The psychological process of reasoning as shown by these experiments does not follow the form of the syllogism, which is not even frequently employed to test the correctness of the conclusion. This is by no means to say that much, or even all, of the reasoning employed resists restatement in syllogistic form. The logical forms are like mathematical symbols in the range of their applicability and also in their abstract and unpictorial character. As well expect to find in 3.14159 a concrete picture of a circle and its diameter as to find in a syllogistic statement a reconstruction of the psychic processes that it symbolizes.

2. The processes are, so far as could be discovered, the familiar processes of association and apperception working under the special conditions of attention to the particular matter in hand.

3. Familiarity with the relations results in quicker solutions and in solutions that are more direct.

4. The established habits of procedure have an influence on the method of attack. This was so strongly marked in some cases as to cause a long use of the "trial and error" method when a general principle was plainly indicated by the data given. Dr. E. H. Lindley found a similar result in the case of children in his "Study of Puzzles."

5. The almost successful solution of a problem often produced mental "cramp." Because the solution was so nearly correct, the mind refused to try a new plan or to make the change of a single figure.

6. There is evidence for believing that reasoning is often an obscure insight or only a guess with an appeal for assistance to the method of "trial and error."

7. Some subjects stated that there was present a vague back-

ground which guided the associations. In some instances the general notion in the background was wrong, and this led to wrong procedure and to a long delay in the solution.

But what is the nature of this "background" or "mental set," and what influence has it on the reasoning process? Is the "mental set" anything more than a neural state brought into activity by seeing the problem? Perhaps we could say that there is a general arithmetical "principle" persisting in the "background" which determines the associations, but again, what is the nature of this general "principle?" Can it be anything more than a general habit of attention under certain circumstances?

The same problem may be put in this way. Reasoning may be defined as a series of successive limitations put on attention and association. But once more, what factors determine this process of limitation? We may say, in general, that it is determined by two things: By something *without*,—in this case the figures arranged in such a way as to form a problem. By something *within*,—the general knowledge of arithmetic and numerical relations and the momentary interest in the solution of the problem. These two influences acting jointly determine the "mental set," which in turn controls our attention and decides what course the associations shall take. But even this does not give us the final word regarding the nature of the "background" or "mental set." For the present this question must be left open.

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XX. ON THE READING AND MEMORIZING OF MEANINGLESS SYLLABLES PRESENTED AT IRREGULAR TIME INTERVALS.

By MARGARET K. SMITH, Ph. D.

The natural tendency to read disconnected syllables in rhythmic groups and the comparative ease of memorizing rhythmic material are matters of common observation. In the laboratory studies made with meaningless syllables, it has been found practically impossible to avoid metrical groupings even when the syllables themselves are presented in strictly uniform series. A subjective tendency toward grouping forces upon them a metrical form which does not exist in their objective arrangement. During the writer's experiments upon Rhythm and Work[1] certain tests were made with the reading of meaningless syllables, in the course of which the question arose as to the extent to which objective inequalities may exist and a rhythmic grouping still be maintained, and, as a secondary question, what the rhythm secured under such difficulties signifies to the individual.

With a view to answering the above questions in connection with voice rhythm, at least, a series of experiments was made in the Psychological Laboratory at Clark University, Worcester, Mass., in the winter of 1900–1901, and though the experiments were unsuccessful as regards finding any order of syllables so irregular as to be wholly refractory to the rhythmic impulse of all the subjects, it seems worth while to give a brief account of them[2]

[1]Rhythmus und Arbeit: Wundt's *Philos. Studien*, Bd. XVI, 1900, 71-133.

[2]The work was under the general direction of Professor Sanford, to whom and to President Hall, the writer wishes to express her sincere thanks for many helpful suggestions, as well as for the many privileges of the laboratories, the library, etc.